The cost of improving and maintaining reliability can be minimized by a model that quantifies the relationships between product cost-effectiveness and availability.

A repairable component (equipment, subsystem) is characterized by its availability, i.e., the ability of the item to perform its required function at or over a stated period of time. Availability can be defined also as the probability that the item (piece of equipment, system) is available to the user, when needed. A large and a complex system or a complicated piece of equipment that is supposed to be available to users for a long period of time (e.g., a switching system or a highly complex communication/transmission system, whose “end-to-end reliability,” including the performance of the software, is important), is characterized by an “operational availability.” This is defined as the probability that the system is available today and will be available to the user in the foreseeable future for the given period of time (see, e.g., Suhir[1]). High availability can be assured by the most effective combination of the adequate dependability (probability of non-failure) and repairability (probability that a failure, if any, is swiftly and effectively removed). Availability of a consumer product determines, to a great extent, customer satisfaction.

Intuitively, it is clear that the total reliability cost, defined as the sum of the cost for improving reliability and the cost of removing failures (repair), can be minimized, having in mind that the first cost category increases and the second cost category decreases with an increase in the reliability level **(Figure 1)**[2]. The objective of the analysis that follows is to quantify such an intuitively more or less obvious relationship and to show that the total cost of improving and maintaining reliability can be minimized.

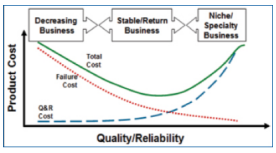

FIGURE 1. How total product cost can be minimized.

Availability index. In the theory of reliability of repairable items, one can consider failures and restorations (repairs) as a flow of events that starts at random moments of time and lasts for random durations of time. Let us assume that failures are rare events, that the process of failures and restorations is characterized by a constant failure rate λ (steady-state portion of the bathtub curve), that the probability of occurrence of n failures during the time t follows the Poisson’s distribution

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \quad (1)$$

(see, e.g., Suhir[1]), that the restoration time *t* is an exponentially distributed random variable, so that its probability density distribution function is

$$f(t) = \mu e^{-\mu t}$$

(2)

where the intensity

$$\mu = \frac{1}{\langle t \rangle}$$

of the restoration process is reciprocal to the mean value of the process. The distribution (2) is particularly applicable when the restorations are carried out swiftly, and the number of restorations (repairs) reduces when their duration increases.

Let *K(t)* be the probability that the product is in the working condition, and *k(t)* is the probability that it is in the idle condition. When considering random processes with discrete states and continuous time, it is assumed that the transitions of the system
S
from the state
si
to the state
sj
are defined by transition probabilities

λij
. If the governing flow of events is of Poisson's type, the random process is a Markovian process, and the probability of state pi(t) = P{S(t) = si,} i = 1,2...,n of such a process, i.e., the probability that the system
S
is in the state
si
at the moment of time
t
, can be found from the Kolmogorov's equation (see, e.g., Suhir
[1]
)

$$\frac{dp_i(t)}{dt} = \sum_{j=1}^{n} \lambda_{ji} p_j(t) - p_i(t) \sum_{j=1}^{n} \lambda_{ij}\ ,\ i = 1,2,\ldots,n$$
(3)

Applying this equation to the processes (1) and (2), one can obtain the following equations for the probabilities *K(t)* and *k(t)*:

$$\frac{dK(t)}{dt} = \mu k(t) - \lambda K(t), \quad \frac{dk(t)}{dt} = \lambda K(t) - \mu k(t).$$
(4)

The probability normalization condition requires that the relationship *K(t)* + *k(t)* =1 takes place for any moment of time. Then the probabilities
K(t)
and
k(t)
in the equations (4) can be separated:

$$\frac{1}{\lambda+\mu}\frac{dK(t)}{dt} + K(t) = \frac{\mu}{\lambda+\mu}, \quad \frac{1}{\lambda+\mu}\frac{dk(t)}{dt} + k(t) = \frac{\lambda}{\lambda+\mu}.$$
(5)

These equations have the following solutions:

$$K(t) = C\exp[-(\lambda+\mu)t] + \frac{\mu}{\lambda+\mu}, \quad k(t) = C\exp[-(\lambda+\mu)t] + \frac{\lambda}{\lambda+\mu}$$
(6)

The constant *C* of integration is determined from the initial conditions, depending on whether the item is in the working or in the idle condition at the initial moment of time. If it is in the working condition, the initial conditions *K*(0) = 1 and *k*(0) = 0 should be used, and

$$C = \frac{\lambda}{\lambda + \mu}.$$

If the item is in the idle condition, the initial conditions $K(0) = 0$ and $k(0) = 1$ should be used,

$$C = \frac{\mu}{\lambda + \mu}.$$

Thus, the availability function can be expressed as

$$K(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu}\exp[-(\lambda + \mu)t],$$

(7) if the item is in the working condition at the initial moment of time, and as

$$K(t) = \frac{\mu}{\lambda + \mu} - \frac{\mu}{\lambda + \mu}\exp[-(\lambda + \mu)t],$$

(8) if the item is idle at the initial moment of time. The constant part

$$K = \frac{\mu}{\lambda + \mu} = \frac{1}{1 + \frac{\lambda}{\mu}} = \frac{1}{1 + \frac{\prec t_r \succ}{\prec t_f \succ}} = \frac{1}{1 + \frac{MTTR}{MTTF}}$$

(9) which is the (available) availability index of the item (9) is the percentage of time,

$$MTTF = \prec t_f \succ = \frac{1}{\lambda}$$

is the mean time to failure, and

$$MTTR = \prec t_r \succ = \frac{1}{\mu}$$

is the averaged time for repair. If the system consists of many items, the formula (9) can be

$$K = \frac{1}{1 + \sum_{i=1}^{n} \frac{\prec t_r \succ_i}{\prec t_f \succ_i}}$$

Minimized reliability costs. It is essential that the cost of achieving and improving reliability

$$C_R = C_R(0)\exp[r(R - R_0)]$$

(10) where R is the reliability level, assessed, e.g., by the actual level of the MTTF; R_0 is the reliability level assessed by an expert; r is a positive factor; and $C_R(0)$ is the cost of the reliability that is the cost of the reliability (12) reflects an assumption that the cost of reliability C_R is its minimum

(14) when the minimization condition

$$C_{\min} = C_R\left(1 + \frac{r}{f}\right) = C_F\left(1 + \frac{f}{r}\right),$$

$$rC_R = fC_F$$

is satisfied. Here f is the factor of the reliability improvement cost, C_F is the cost of the reliability deterioration caused by the failure, and r is the factor of the reliability deterioration caused inversely proportional to the MTTR. The formula (14) yields

$$C_{\min} = \frac{C_R}{\frac{C_F}{C_R}} = \frac{C_F}{\frac{C_F}{C_R} + 1}.$$

The availability function can be established by the equation (9). The formula (13) shows that

$$\frac{C_F}{C_R} = \frac{1}{\frac{r}{f}} - 1.$$

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